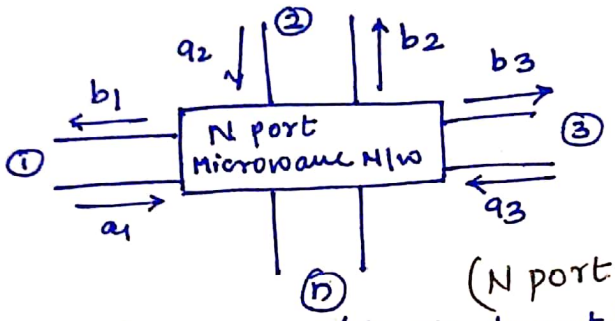
SUBJECT: Microwave and Radar Engineering

Scattering Parameters (S-MATRIX)

At low frequency, circuit can be represented as two port network and characterize by their parameters such as z-parameter, Y Parameters and h-parameter. These parameters relate total voltages and currents at two port network. If the Frequency are in microwave range, then h, Y, Z parameter's can not be used. So in microwave analysis - Power (analysis) relationship between various ports of microwave function, is defined in terms of scattering parameters / S-MATRIX.



(N port microwave function)

Microwave function is an multiport junction as shown above.

a_n : amplitude of incident wave of nth port.

b_n : amplitude of reflected wave of nth port.

→ S Matrix is defined in terms of these incident and reflected voltages as —

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \cdots S_{1n} \\ S_{21} & S_{22} \cdots S_{2n} \\ \vdots & \\ S_{n1} & S_{n2} \cdots S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Reflected voltage/wave output — S Matrix — Input wave/voltage

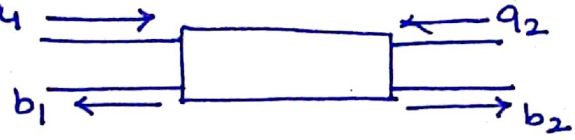
or, simply. $[b] = [S][a]$

Scattering coefficient — $\boxed{S_{ij} = \frac{b_i}{a_j}}$

Note: All ports of microwave function except incident should be terminated to matched load to avoid reflection.

- For 2 port microwave function –

i/p a_1 → ← a_2 o/p



$\because$ $[b] = [S][a]$

$\therefore$ $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ or, $b_1 = S_{11}a_1 + S_{12}a_2$ —i)

$b_2 = S_{21}a_1 + S_{22}a_2$ —ii)

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$; $S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$; $S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$

Properties of S-matrix : (1) It is a square matrix of size $n \times n$ where n represent number of ports.

(2) Principle diagonal of S-matrix reprent reflection coffficient of respective ports.

if $S_{11} = S_{22} = S_{33} = S_{44} = 0$, then all 4 ports are perfectly matched.

(3) symmetry Property :- for a perfectly matched and reciprocal n/w the S-matrix is symmetrical i.e.

$$S_{ij} = S_{ji}$$

(4) Zero property → The sum of product of each term of any row (or columns) multiplied by the conjugate (complex conjugate) of the corresponding terms of any other row (or columns) is zero.

$$\sum_{n=1}^{N} S_{nK} \cdot S_{nj}^{*} = 0 \quad (K \neq j)$$

i.e. $R_1 R_2^* = 0$

$C_1 C_2^* = 0$

(5) Unity Property → The sum of the products of each term of any row/columns multiplied by its complex conjugate is unity. i.e.

$$[S][S^*] = I$$

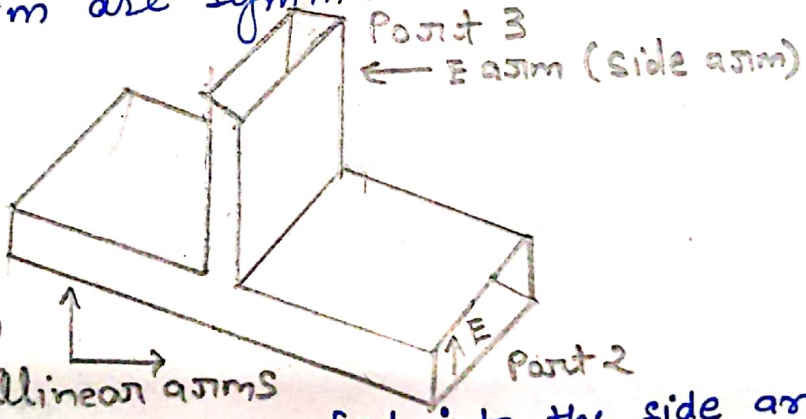
TOPIC: 1. E Plane Tee 2. H plane Tee 3. Magic Tee

But before discussing these topics we must know what are microwave Tees?

These are serial microwave component which are used for proper Tx^n of signal. Microwave Junction(devices) used to connect two or more microwave device. Same as microwave Tees are interconnecting circuit formed in the shape of english alphabet 'T'.

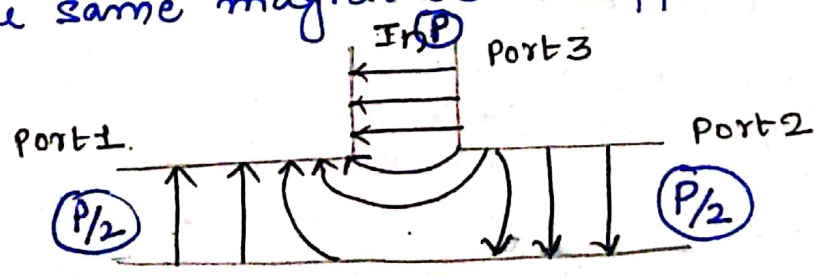
(1) E Plane Tee / series Tee:

E Plane Tee is a waveguide Tee in which the axis of its E-arm (side arm) is parallel to E field of main guide as in figure (1). It have 3 arms. Port 1 and Port 2 are called collinear Arm and third one is E arm / side arm. The input signal can be fed into any of the ports. collinear arm are symmetrical about the side/E arm.

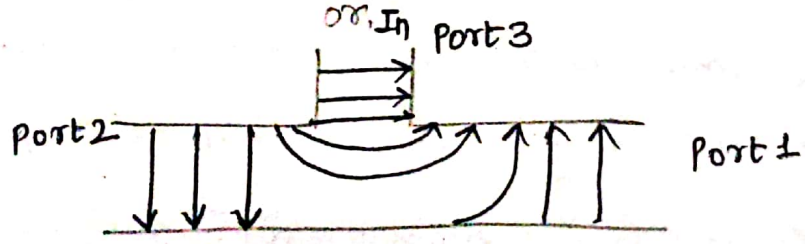


(fig. 6.2(a) E-Plane Tee)

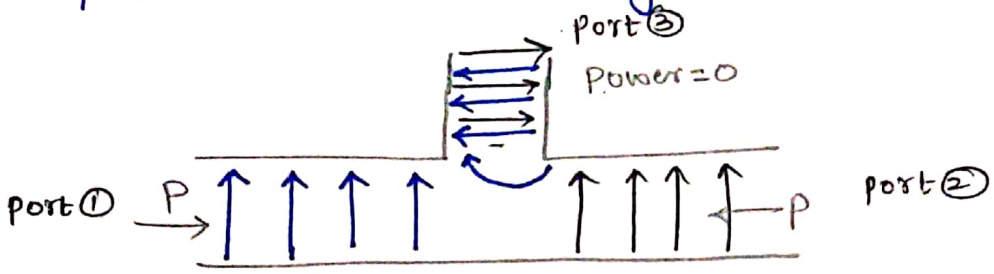
→ (i) when the input waves are fed into the side arm, the waves appearing at PORT 1 and PORT 2 of collinear arm will have same magnitude but opposite phase.



$S_{13} = -S_{23}$



→ When input waves are fed through main arm (port ① and ②)



Here port 3 is called subtracting arm.

S-Matrix: $[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}_{3\times3}$

∵ port 3 is perfectly matched, $S_{33} = 0$

S_{13} : input from Port 3, output at port 1.

S_{23} : " " " " " " port 2.

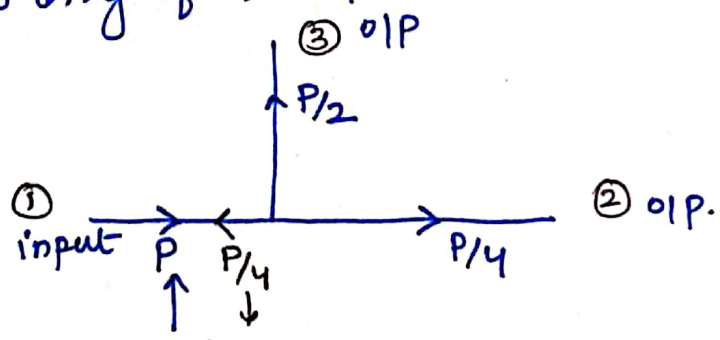
we know, $\boxed{S_{13} = -S_{23}}$

$\boxed{S_{12} = S_{21}}$ (symmetry property)

$$S_{13} = \sqrt{\frac{P/2}{P}} = 1/\sqrt{2} = -S_{23} \quad \text{---- (i)}$$

for $S_{11}, S_{22}, S_{12}, S_{21}$ →

let input wave fed to any of main port, (Port 1 / Port 2)



(Reflected back because port 1 and 2 are not perfectly matched)

$$S_{12} = \sqrt{\frac{P/4}{P}} = \frac{1}{2}$$

$$S_{21} = \sqrt{\frac{P/4}{P}} = \frac{1}{2}$$

$$S_{11} = \sqrt{\frac{P/4}{P}} = \frac{1}{2}$$

$$S_{22} = \sqrt{\frac{P/4}{P}} = \frac{1}{2}$$

So, S-matrix of E plane Tee –

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}_{3\times3}$$

H-Plane Tee (Shunt Tee)

H-plane Tee is a waveguide in which axis of the side arm is parallel to the planes of magnetic field of main waveguide. (Figure 2(a)). As all the three arms lie in the plane of magnetic field. The magnetic field divide itself into arms, thus it's a Shunt Tee.

* H line divided → current divided → thus it's called Shunt Junction.

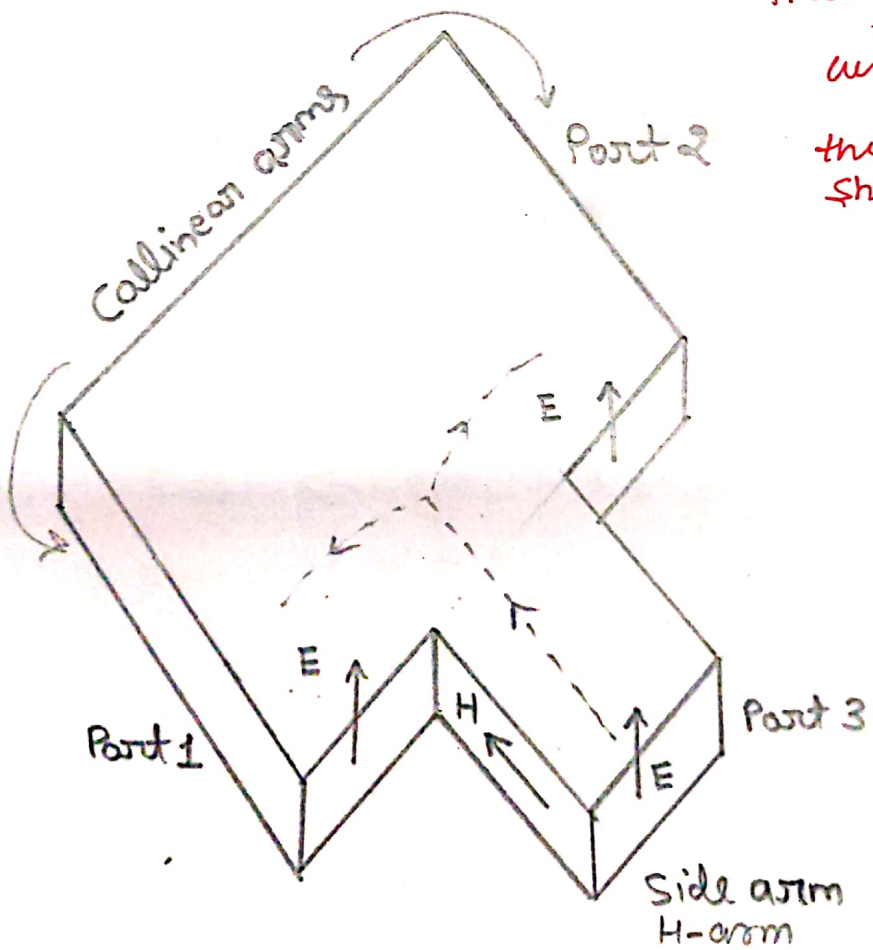


fig. 2(a) H- Plane tee

H-Plane is completly symmetrical.

i) if the input is fed into port 3 (H-arm), the wave will split equally into port 1 and port 2 with some magnitude and in phase. (fig 2(b))

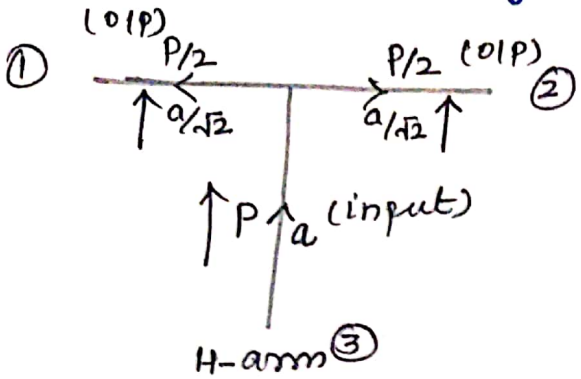


fig 2(b)

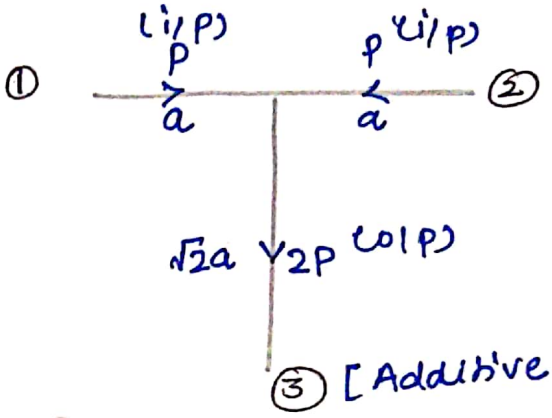


fig 2(c)

Here, a → voltage
P → power,
when, R = 1, $P = a^2$

② If two input waves are fed into port 1 and port 2 of colline arm, the output wave at port 3 will be in phase and additive. In this case maximum energy delievered.

S-matrix:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}_{3\times3}$$

as H-plane is symmetrical—

$$S_{12} = S_{21}$$
$$S_{13} = S_{31}$$
$$S_{23} = S_{32}$$

So,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}_{3\times3}$$

since, port3 is perfectly matched- $S_{33} = 0$

S_{13} → input form port 3 and output at port 1. (fig (2b))

$S_{13} = \sqrt{\frac{P/2}{P}} = 1/\sqrt{2}$; similary $S_{23} = \sqrt{\frac{P/2}{P}} = 1/\sqrt{2}$

for. S_{11}, S_{12}, S_{22} using unitary and uso property —

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} 3\times3.$$

MAGIC TEE (E-H plane Tee)

from E-plane and H-plane Tee we conclude that a 3 port D/w cannot be lossless, reciprocal and matched at all ports. In magic Tee there are four ports and all 4 ports are perfectly matched. It is the magic in magic Tee.

ie. $\boxed{S_{11} = S_{22} = S_{33} = S_{44} = 0}$ for magic Tee.

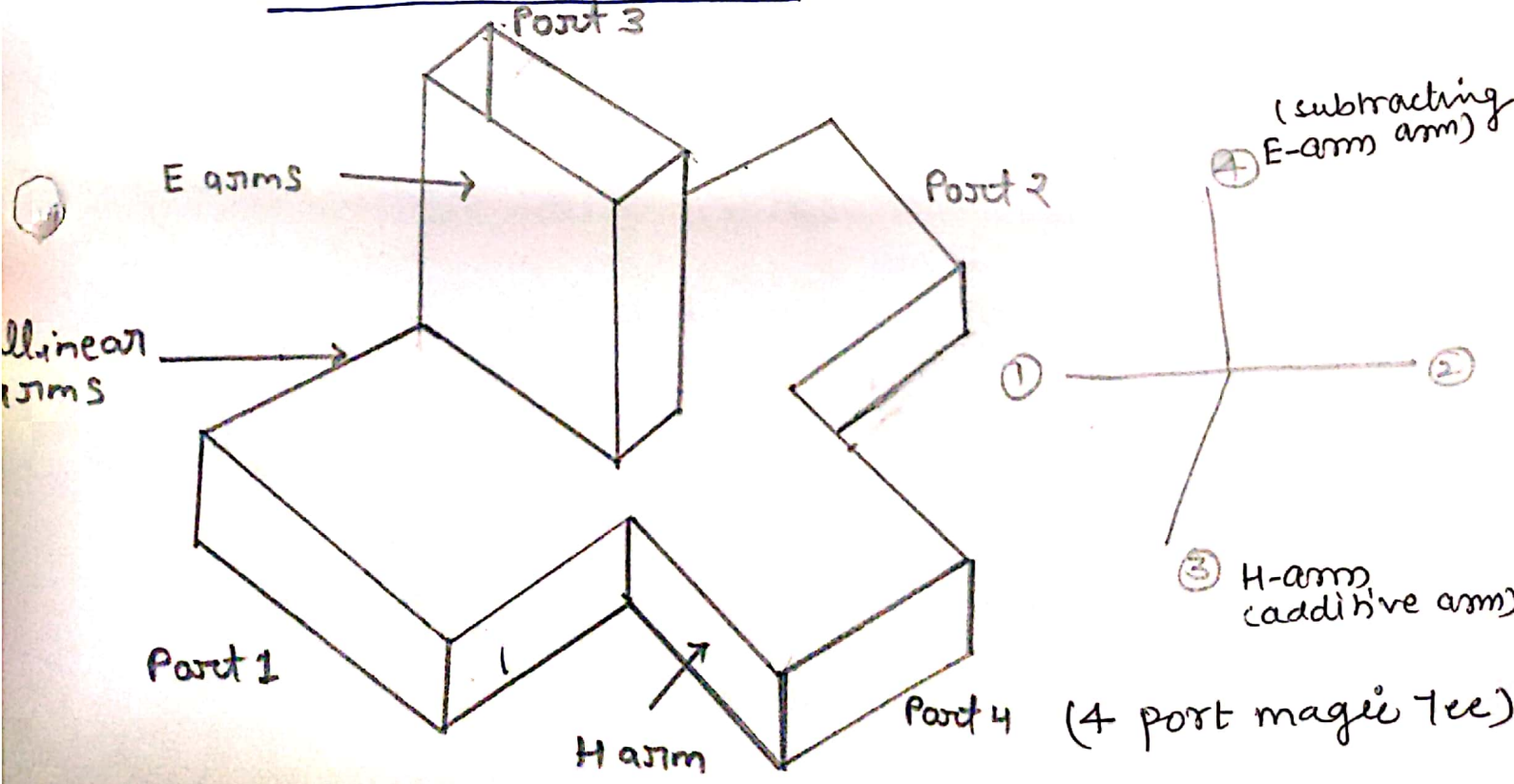


fig. 6.4(a) Magic Tee

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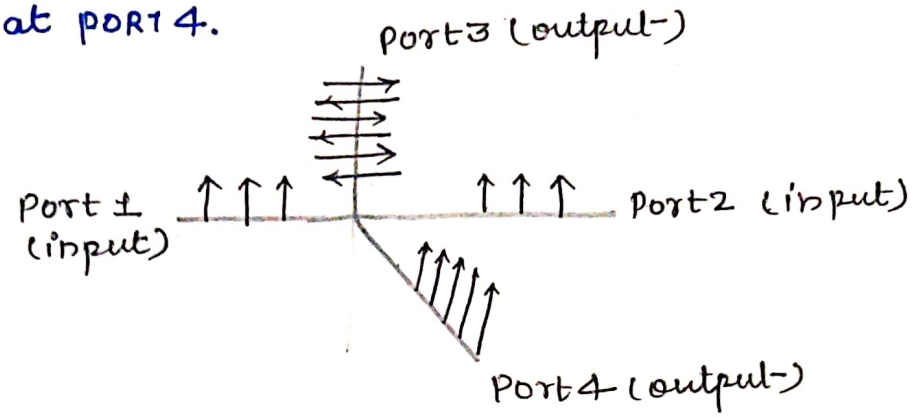
Magic Tee has following characteristic-

(i) When input applied at port 1 and port 2 : If two wave of equal magnitude and same phase are fed into Port 1 and Port 2, then output will be zero at port 3 (E-arm/subtracting arm) and additive at PORT 4.



(ii) If a wave is fed into port 4 (H-arm), it will equally divided between Port 1 and Port 2 of collinear arms and will not appear at port 3 (E-arm). [fig (i)]

(iii) If a wave is fed into Port 3 (E-arm), it will produce equal magnitude but opposite in phase at port 1 and port 2, output at port 4 is zero. [fig (ii)]

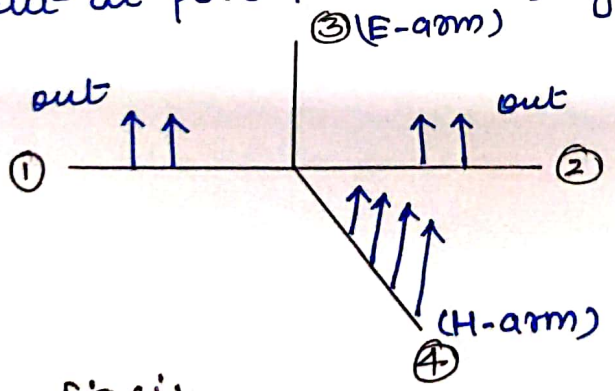


fig (i)

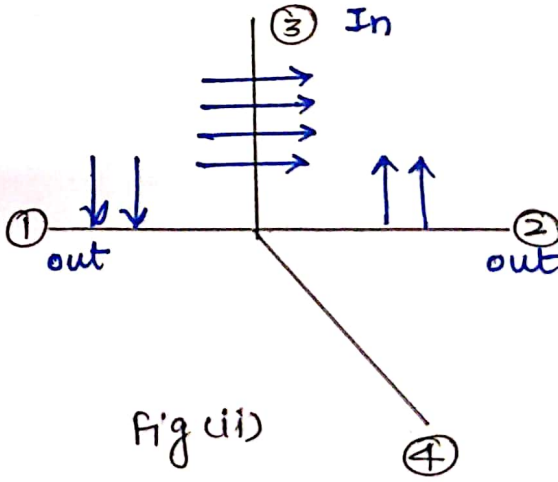


fig (ii)

Note: In magic Tee E-arm and H-arm are isolated [$S_{34} = S_{43} = 0$]
* and collinear arm (Port 1, Port 2) are isolated [$S_{12} = S_{21} = 0$]

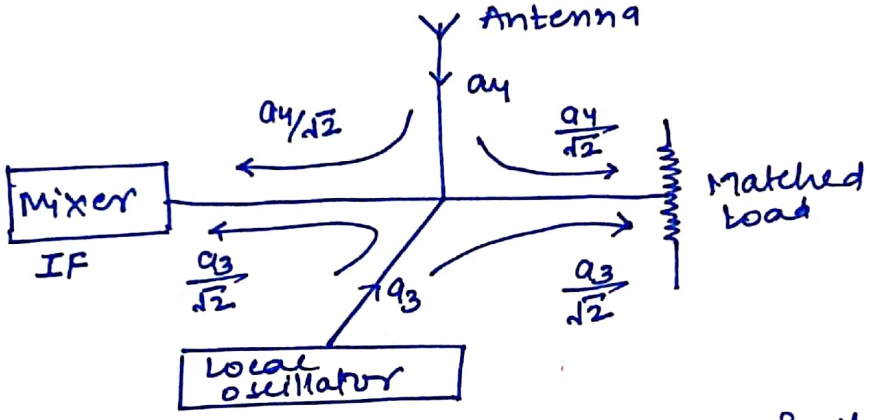
S-matrix for magic Tee-

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}_{4\times4}$$

* If a wave is fed into one of collinear arms at PORT 1 or PORT 2, it will not appear in the other collinear arm at PORT 2 or PORT 1

Application of Magic Tee →

(1) Magic Tee as a mixer: Magic tee can also be used in microwave receiver as mixer, where signal and local oscillator output are fed into E and H-arm respectively.



Half of local oscillator power and half of the received power from antenna goes to mixer where they are raise to generate IF.